NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science ; Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BSAM	**LEVEL: 6**
COURSE CODE: ODE602S	**COURSE NAME:** ORDINARY DIFFERENTIAL EQUATIONS
SESSION: JANUARY 2023	**PAPER: THEORY**
DURATION: 3 HOURS	**MARKS: 80**

SUPPLEMENTARY / SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	**Prof A.S EEGUNJOBI**
MODERATOR:	Prof S.A REJU

INSTRUCTIONS

1. Answer ANY FOUR(4) questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

1. Solve the following initial value problems:

   (a)
$$\frac{x^2y'(x)}{5} + x^3y(x) = \frac{e^{-x}}{5}, \quad y(-1) = 0, \quad \text{for} \quad x < 0$$
(5)

   (b)
$$y'(x)\sin x + y(x)\cos x = 2e^x, \quad y(1) = a, \quad 0 < x < \pi$$
(5)

   (c) If a constant number k of fish are harvested from a fishery per unit time, then a logistic model for the population $P(t)$ of the fishery at time t is given by
$$\frac{dP(t)}{dt} = P(t)(5 - P(t)) - 4, \quad P(0) = P_0$$

      i. Solve the IVP. (5)

      ii. Determine the time when the fishery population becomes quarter of the initial population (5)

2. (a) If y_1 and y_2 are two solutions of second order homogeneous differential equation of the form
$$y''(x) + p(x)y'(x) + q(x)y(x) = f(x)$$
where $p(x)$ and $q(x)$ are continuous on an open interval I, derive the formula for $u(x)$ and $v(x)$ by using variation of parameters. (6)

   (b) If
$$y_1(x) = 2x + 1, \quad W(y_1, y_2) = 2x^2 + 2x + 1, \quad y_2(0) = 0$$
find $y_2(x)$ (7)

   (c) Solve
$$8x^2y''(x) + 16xy'(x) + 2y(x) = 0$$
(7)

3. (a) Solve the Euler equation
$$x^2y''(x) + 15xy'(x) + 58y(x) = 0, \quad y(1) = 1, \quad y'(1) = 0$$
(7)

   (b) Solve the following differential equations by method of variation of parameters $y''(x) + y(x) = \tan x$ (8)

   (c) Solve the following differential equations by method of undetermined coefficients
$$y''(x) + 2y'(x) + 2y(x) = -e^x(5x - 11), y(0) = -1, \quad y'(0) = -3$$
(5)

4. (a) Find the Laplace inverse of

$$\frac{s^2 - 10s + 13}{(s-7)(s^2 - 5s + 6)}$$

(6)

(b) Compute

$$\mathcal{L}^{-1}\left\{\frac{2s^3 + 2s^2 + 4s + 1}{(s^2+1)(s^2+s+1)}\right\}$$

(7)

(c) Solve using Laplace transform

$$y'(t) - 2y(t) = 6t^3 e^{2t}, \qquad y(0) = -3$$

(7)

5. (a) Use reduction of order method to find $y_2(x)$ if

$$x^2 y'' - 3xy' + 4y = 0; \quad y_1(x) = x^2$$

(5)

(b) Find the first five terms in the series solution of

$$y'(x) + y(x) + x^2 y(x) = \sin x, \quad \text{with} \quad y(0) = a.$$

(5)

(c) Use the power series method to solve

$$y''(x) + 4y(x) = 0, \quad y(0) = 1, \quad y'(0) = 2$$

(10)

End of Exam!